Journal of Engineering Physics and Thermophysics, Vol. 76, No. 5, 2003

CONNECTIVITY COEFFICIENTS IN THE CHARACTERISTIC DETERMINANTS OF THE SYSTEMS OF EQUATIONS OF MOTION OF ISOTROPIC THERMOELASTIC MEDIA

S. M. Bosyakov UDC 539.3

Characteristic determinants and characteristic equations have been obtained for the systems of equations of motion of isotropic thermoelastic media in stresses with allowance for the finite velocity of propagation of thermal disturbances. The connectivity coefficients of the mechanical and thermal fields under problems of different dimension have been determined.

Introduction. Fundamental theoretical and practical investigations of the regularities of propagation of thermoelastic waves in generalized thermomechanics have been carried out by many authors. The best known of them are [1–5] devoted to the application of the theory of plane waves to the systems of equations of motion of isotropic and anisotropic media in displacements. However, thermoelastic stress waves possess a number of advantages over displacement waves, which is confirmed by experiments [6]. In the present work, we have studied the regularities of propagation of stress waves in a thermoelastic isotropic medium with the use of the classical characteristic method [7].

Characteristic Determinants. The equations of the dynamic theory of temperature stresses in the case of a homogeneous isotropic body will be obtained from the equations of motion in displacements and the Hooke law (mass forces are absent) [8]

$$\mu\Delta_3 u_i + (\lambda+\mu)\,\partial_i\partial_k u_k = \rho\ddot{u}_i + \beta\partial_i T\,, \quad \sigma_{ij} = (\lambda e_{kk} - \beta T)\,\delta_{ij} + 2\mu e_{ij}\,, \tag{1}$$

where $\mathbf{u} = (u_1, u_2, u_3)$ is the displacement vector, T is the absolute temperature, $e_{ij} = (\partial_i u_j + \partial_j u_i)/2$ is the strain tensor, $\partial_i = \partial/\partial x_i$, and $\Delta_3 = \partial_k^2$; the points denote differentiation with respect to time; summation is carried out over the subscript $k = \overline{1, 3}$; $\delta_{ij} = 0$, when $i \neq j = \overline{1, 3}$, and $\delta_{ij} = 1$, when $i = j = \overline{1, 3}$.

Equations (1) yield the following system of equations:

$$\mu\Delta_3 e_{ij} + (\lambda+\mu)\,\partial_i\partial_j e_{kk} = \rho\ddot{e}_{ij} + \beta\partial_i\partial_j T\,; \tag{2}$$

$$e_{ij} = \left(\sigma_{ij} - \frac{\lambda\sigma_{kk} - 2\mu\beta T}{3\lambda + 2\mu}\,\delta_{ij}\right)\Big/ 2\mu\,, \quad i, j = \overline{1, 3}\,. \tag{3}$$

As a result of the substitution of (3) into (2) we obtain

$$(3\lambda+2\mu)\left(\Delta_3\sigma_{ij} - \frac{\rho\ddot{\sigma}_{ij}}{\mu}\right) - \lambda\left(\Delta_3\sigma_{kk} - \frac{\rho\ddot{\sigma}_{ij}}{\mu}\right)\delta_{ij} + 2\,(\lambda+\mu)\,\partial_i\partial_j\sigma_{kk} = 2\beta\mu\left(\left(\frac{\rho\ddot{T}}{\mu} - \Delta_3 T\right)\delta_{ij} - \partial_i\partial_j T\right), \quad i, j = \overline{1, 3}\,. \tag{4}$$

To close system (4) we add to it the hyperbolic heat-conduction equation [4]

$$K\Delta_3 T - c_v\,(\dot{T} + \tau\ddot{T}) = \beta T_0\,(\dot{e}_{kk} + \tau\ddot{e}_{kk})\,. \tag{5}$$

From (5), with the use of (3), we obtain

Belarusian State University, 4 Skorina Ave., Minsk, 220050, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 5, pp. 138–143, September–October, 2003. Original article submitted February 7, 2003.

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$$K\Delta_3 T - \left(c_v + \frac{3\beta^2 T_0}{3\lambda + 2\mu}\right)(\dot{T} + \tau\ddot{T}) = \beta T_0\,(\dot{\sigma}_{kk} + \tau\ddot{\sigma}_{kk})/(3\lambda + 2\mu)\ , \tag{6}$$

or, taking into account that $\beta = (3\lambda + 2\mu)\alpha_T$ (α_T is the coefficient of linear thermal expansion), we obtain

$$K\Delta_3 T - (c_v + 3\alpha_T^2 T_0\,(3\lambda + 2\mu))\,(\dot{T} + \tau\ddot{T}) = \alpha_T T_0\,(\dot{\sigma}_{kk} + \tau\ddot{\sigma}_{kk})\ . \tag{7}$$

We specify the initial data to system (4) and (7) on the hypersurface $Z(t, x_1, x_2, x_3) = 0$ and pass to new variables according to the following scheme [7]:

$$Z = Z\,(t, x_1, x_2, x_3)\ ,\quad Z_i = Z_i\,(t, x_1, x_2, x_3)\ ,\quad i = \overline{1, 3}\ .$$

We express the derivatives with respect to the previous variables by the derivatives with respect to the new variables and substitute them into (4) and (7):

$$\left((3\lambda + 2\mu)\frac{\partial^2\sigma_{ij}}{\partial Z^2} - \left(\lambda\frac{\partial^2\sigma_{kk}}{\partial Z^2} - 2\beta\mu\frac{\partial^2 T}{\partial Z^2}\right)\delta_{ij}\right)\left(g_3^2 - \frac{\rho p_0^2}{\mu}\right) +$$

$$+ 2p_i p_j\left((\lambda + \mu)\frac{\partial^2\sigma_{kk}}{\partial Z^2} + \beta\mu\frac{\partial^2 T}{\partial Z^2}\right) + \ldots = 0\ ,\quad i, j = \overline{1, 3}\ ,$$

$$\left(Kg_3^2 - \left(c_v + \frac{3\beta^2 T_0}{3\lambda + 2\mu}\right)p_0^2\right)\frac{\partial^2 T}{\partial Z^2} - \frac{\tau\beta T_0 p_0^2}{(3\lambda + 2\mu)}\frac{\partial^2\sigma_{kk}}{\partial Z^2} + \ldots = 0\ ,$$

where $p_0 = \dfrac{\partial Z}{\partial t}$, $p_i = \dfrac{\partial Z}{\partial x_i}$, $i = \overline{1, 3}$, and $g_3^2 = p_k^2$.

The nonlinear differential equation of first order which must be satisfied by the characteristic surface $Z(t, x_1, x_2, x_3) = 0$ of system (4) and (7) will have the form

$$\det\,\|\omega_{ij}\|_{i,j=\overline{1,3}} \times \det\,\|\zeta_{nm}\|_{n,m=\overline{1,4}} = 0\ ,$$

where

$$\omega_{ii} = g_3^2 - \frac{\rho p_0^2}{\mu}\quad \text{(the remaining } \omega_{ij} \text{ are equal zero },\ i, j = \overline{1, 3})\ ,$$

$$\zeta_{nn} = 2\,(\lambda + \mu)\left(g_3^2 + p_n^2 - \frac{\rho p_0^2}{\mu}\right),\quad \zeta_{nm} = 2\,(\lambda + \mu)\,p_n^2 - \lambda\left(g_3^2 - \frac{\rho p_0^2}{\mu}\right),\quad \xi_{4n} = -\,\alpha_T T_0 \tau p_0^2\ ,$$

$$\zeta_{n4} = 2\beta\mu\left(g_3^2 - \frac{\rho p_0^2}{\mu} + p_n^2\right),\quad \zeta_{44} = Kg_3^2 - \tau p_0^2\,(c_v + 3\alpha_T^2 T_0\,(3\lambda + 2\mu))\ ,\quad n \neq m = \overline{1, 3}\ .$$

The equality of the determinant $\det\,\|\omega_{ij}\|_{i,j=\overline{1,3}}$ to zero yields the existence of three discontinuity surfaces propagating with the same velocity $V = p_0/g_3 = \sqrt{\mu/\rho}$, which is equal to the velocity of propagation of a transverse elastic wave c_2. After simple transformations, the equality of the determinant $\det\,\|\zeta_{nm}\|_{n,m=\overline{1,4}}$ to zero will be written as follows:

TABLE 1. Values of the Thermomechanical Parameters

Thermomechanical quantity	Materials			
	aluminum	copper	steel	lead
c_1, m/sec	6260	4700	5800	2160
c_2, m/sec	3080	2260	2530	700
K, W/(m·deg)	207	400	57	35
ε_3	0.0526	0.0243	0.0153	0.0852
ε_2	0.0470	0.0219	0.0141	0.0819
ε_1	0.0356	0.0168	0.0114	0.0733
ω_*, GHz [1]	466	173	175	191

$$\begin{vmatrix} 2(1-a^2)\times\left(1+n_1^2-\frac{v^2}{a^2}\right), & 2(1-a^2)\,n_1^2-(1-2a^2)\left(1-\frac{v^2}{a^2}\right), & 2(1-a^2)\,n_1^2-(1-2a^2)\left(1-\frac{v^2}{a^2}\right), & \frac{2}{3}\left((1+n_1^2)\,a^2-v^2\right), \\ 2(1-a^2)\,n_2^2-(1-2a^2)\left(1-\frac{v^2}{a^2}\right), & 2(1-a^2)\times\left(1+n_2^2-\frac{v^2}{a^2}\right), & 2(1-a^2)\,n_2^2-(1-2a^2)\left(1-\frac{v^2}{a^2}\right), & \frac{2}{3}\left((1+n_2^2)\,a^2-v^2\right), \\ 2(1-a^2)\,n_3^2-(1-2a^2)\left(1-\frac{v^2}{a^2}\right), & 2(1-a^2)\,n_3^2-(1-2a^2)\left(1-\frac{v^2}{a^2}\right), & 2(1-a^2)\times\left(1+n_3^2-\frac{v^2}{a^2}\right), & \frac{2}{3}\left((1+n_3^2)\,a^2-v^2\right), \\ -\varepsilon_3 n_* v^2, & -\varepsilon_3 n_* v^2, & -\varepsilon_3 n_* v^2, & 1-n_* v^2\,(1+\varepsilon_3), \end{vmatrix} = 0 . \qquad (8)$$

Here $a = c_2/c_1$ is the ratio of the velocities of propagation of the longitudinal and transverse waves, $c_1 = \sqrt{(\lambda+2\mu)/\rho}$, $n_* = \tau\omega_*$ is the characteristic number of vibrations, $\omega_* = c_v c_1^2/K$ is the characteristic quantity having the dimension of frequency, $v = V/c_1$ is the dimensionless velocity of propagation of the discontinuity surface, $n_i = \cos\alpha_i = p_i/g$ are the direction cosines of the normal to the characteristic surface, $i = \overline{1,3}$, and $\varepsilon_3 = 3\beta^2 T_0/(3c_1^2-4c_2^2)C_v = 3\alpha_T^2 T\,(3\lambda+2\mu)/C_v$ is the connectivity coefficient for the three-dimensional interconnected problem of thermoelasticity (dimensionless quantity dependent on the thermal and mechanical properties of material), $C_v = c_v\rho$.

The values of the connectivity coefficient ε_3 which have been calculated from the numerical data of [9, 10] are given for certain structural materials at a temperature of 20^{o}C in Table 1.

Let us consider a plane dynamic problem of generalized interconnected thermoelasticity under plane strain and take $e_{33} = 0$ for the sake of definiteness. In this case, the equations of motion of an isotropic medium in stresses can be obtained either from (2) and (5) by the substitution of (3) or by the substitution of the expression $\sigma_{33} = (\lambda\,(\sigma_{11}+\sigma_{22}) - 2\mu\beta T/2\ (\lambda+\mu)$ into (4) and (6). After standard transformations, we obtain a system of four differential equations for three independent components of the stresses σ_{11}, $\sigma_{12} = \sigma_{21}$, and σ_{33} and the temperature T:

$$(\lambda+2\mu)\left(\Delta_2\sigma_{ii}-\frac{\rho\ddot{\sigma}_{ii}}{\mu}\right)-\lambda\left(\Delta_2\sigma_{jj}-\frac{\rho\ddot{\sigma}_{ii}}{\mu}\right)+$$

$$+\,2\,(\lambda+\mu)\,\partial_i^2\,(\sigma_{11}+\sigma_{22}) = 2\beta\,(\rho\ddot{T}-\mu\Delta_2 T)\,,\quad i\neq j = 1, 2\,,$$

$$\mu\Delta_2\sigma_{12} - \rho\ddot{\sigma}_{12} + \mu\partial_1\partial_2\,(\sigma_{11} + \sigma_{22}) = 0\,, \tag{9}$$

$$K\Delta_2 T - (\dot{T} - \tau\ddot{T})\left(c_v + \frac{\beta^2 T}{\lambda + \mu}\right) = \beta T_0\,(\dot{\sigma}_{11} + \dot{\sigma}_{22} + \tau\,(\ddot{\sigma}_{11} + \ddot{\sigma}_{22}))/2\,(\lambda + \mu)\,, \quad \Delta_2 = \partial_1^2 + \partial_2^2\,.$$

We specify the initial data to system (9) on the hyperplane $Z(t,\ x_1, x_2) = 0$, replace the variables, and substitute the derivatives with respect to the new variables into (9):

$$\left(g_2^2 - \frac{\rho p_0^2}{\mu}\right)\left((\lambda + 2\mu)\,\frac{\partial^2\sigma_{ii}}{\partial Z^2} - \lambda\,\frac{\partial^2\sigma_{jj}}{\partial Z^2} + 2\beta\mu\,\frac{\partial^2 T}{\partial Z^2}\right) +$$

$$+\,2\,(\lambda + \mu)\,p_i^2\left(\frac{\partial^2\sigma_{11}}{\partial Z^2} + \frac{\partial^2\sigma_{22}}{\partial Z^2}\right) + \ldots = 0\,, \quad g_2^2 = p_1^2 + p_2^2\,, \quad i \neq j = 1, 2\,;$$

$$(\mu g_2^2 - \rho p_0^2)\,\frac{\partial^2\sigma_{12}}{\partial Z^2} + \mu p_1 p_2\left(\frac{\partial^2\sigma_{11}}{\partial Z^2} + \frac{\partial^2\sigma_{22}}{\partial Z^2}\right) = 0\,; \tag{10}$$

$$K g_2^2 - \tau p_0^2\left(c_v + \frac{\beta^2 T}{\lambda + \mu}\right)\frac{\partial^2 T}{\partial Z^2} = \frac{\beta T_0 p_0^2\tau}{2\,(\lambda + \mu)}\left(\frac{\partial^2\sigma_{11}}{\partial Z^2} + \frac{\partial^2\sigma_{22}}{\partial Z^2}\right).$$

The equation of the characteristic hyperplane $Z(t,\ x_1, x_2) = 0$ of system (9) will be written as the equality to zero of the determinant whose components are the coefficients of the partial derivatives of second order in Z in (10). After simple transformations, we obtain

$$\begin{vmatrix} 1 - \dfrac{v^2}{a^2} + 2\,(1 - a^2)\,n_1^2\,, & 2\,(1 - a^2)\,n_1^2 - -(1 - 2a^2)\left(1 - \dfrac{v^2}{a^2}\right), & 0\,, & a^2 - v^2\,, \\ 2\,(1 - a^2)\,n_2^2 - -(1 - 2a^2)\left(1 - \dfrac{v^2}{a^2}\right), & 1 - \dfrac{v^2}{a^2} + 2\,(1 - a^2)\,n_2^2\,, & 0\,, & a^2 - v^2\,, \\ a^2 n_1 n_2\,, & a^2 n_1 n_2\,, & a^2 - v^2\,, & 0\,, \\ -\,\varepsilon_2 n_* v^2\,, & -\,\varepsilon_2 n_* v^2\,, & 0\,, & 1 - n_* v^2\,(1 + \varepsilon_2)\,, \end{vmatrix} = 0\,, \tag{11}$$

where $\varepsilon_2 = \beta^2 T_0/C_v\ (c_1^2 - c_2^2)$ is the connectivity coefficient for the two-dimensional problem of interconnected thermoelasticity. The values of ε_2 for four materials at a temperature of 20°C are given in Table 1.

The system of equations of the interconnected dynamic problem of thermoelasticity allows solutions dependent on time and on one spatial coordinate and independent of the other coordinates. Thus, for example, in [3] a study is made of the problems of dispersion and damping in the case of one-dimensional modes of propagation of plane waves.

We will assume that motion occurs along the axis $x_1 \equiv x$. In this case the stress tensor is characterized by one independent component $\sigma_{11} = (\lambda + 2\mu)e_{11} - \beta T$; $\sigma_{22} = \sigma_{33} = (\lambda\sigma_{11} - 2\mu\beta T)/(\lambda + 2\mu)$ and its other components are equal to zero. Then from (4) and (7) we obtain

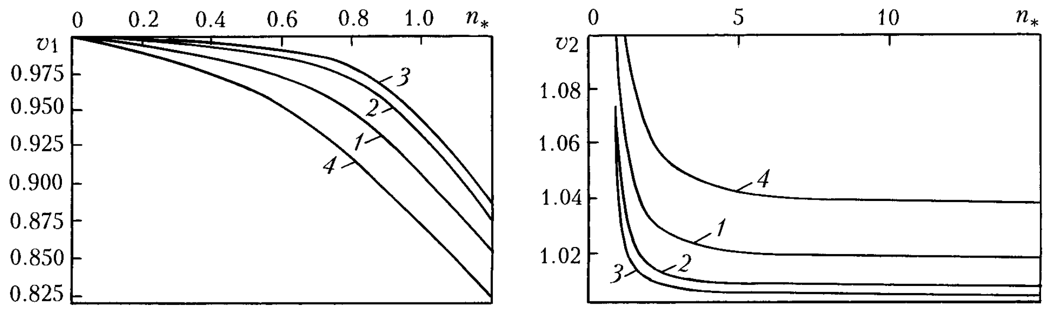


Fig. 1. Velocities $v_{1,2}$ as functions of the parameter n_*: 1) aluminum; 2) copper; 3) steel; 4) lead.

$$c_1^2\partial_1^2\sigma_{11} = \ddot{\sigma}_{11} + \frac{\beta\ddot{T}}{\rho}\,, \quad K\partial_1^2 T - \left(c_v + \frac{\beta^2 T_0}{\lambda + 2\mu}\right)(\dot{T} + \tau\ddot{T}) = \beta T_0\,(\dot{\sigma} + \tau\ddot{\sigma})/(\lambda + 2\mu)\,. \tag{12}$$

The equation of the characteristic plane $Z(t, x_1) = 0$ of system (12) will be written as follows ($g_1^2 = p_1^2$):

$$\begin{vmatrix} c_1^2 g_1^2 = p_0^2\,, & -\beta p_0^2/\rho\,, \\ -\beta T_0 \tau p_0^2/(\lambda + 2\mu)\,, & K g_1^2 - \left(c_v + \dfrac{\beta^2 T_0}{\lambda + 2\mu}\right) p_0^2 \end{vmatrix} = 0\,.$$

Hence

$$\begin{vmatrix} 1 - v^2\,, & -v^2\,, \\ -\varepsilon_1 n_* v^2\,, & 1 - n_* v^2\,(1 + \varepsilon_1) \end{vmatrix} = 0\,, \tag{13}$$

where $\varepsilon_1 = \beta^2 T_0/C_v c_1^2$ is the connectivity coefficient for the interconnected one-dimensional dynamic problems of thermoelasticity. We note that ε_1 is no different from the connectivity coefficient ε (adopted in the theory of plane harmonic thermoelastic displacement waves) for an isotropic thermoelastic body [1] (the values of $\varepsilon_1 = \varepsilon$ are given in Table 1). The coefficient $\varepsilon = \beta^2 T_0/(c_v A_1)$ ($A_1 = \lambda + 2\mu$ is the elasticity constant) is universally used in investigations of the regularities of propagation of thermoelastic waves in both isotropic and anisotropic media irrespective of the dimension of the problem. However, as follows from the above computations, the connectivity coefficients in the dynamic problems of generalized thermomechanics in stresses are dissimilar; the connectivity coefficient increases with dimension. We also note that another characteristic quantity, $\omega_* = c_1^2 c_v/K$, has one and the same form irrespective of the dimension of the system of equations of motion.

Solutions of the Characteristic Equations. Let us expand determinant (11):

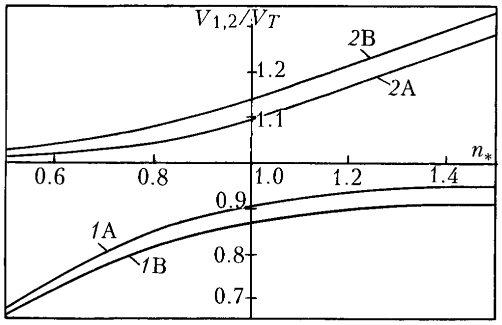
$$(a^2 - v^2)^2\,(n_* v^2 - v^2\,(1 + n_* + n_*\varepsilon_2 - a^2 n_*\varepsilon_2) + 1) = 0\,. \tag{14}$$

This yields the existence of two thermoelastic wave propagating with velocities $V_1 = c_1 v_1$ and $V_2 = c_1 v_2$, where the dimensionless velocities $v_{1,2}$ are determined by the following expressions:

$$v_{1,2} = \sqrt{(B_2 \mp \sqrt{B_2^2 - n_*}\,)/n_*}\,, \quad 2B_2 = 1 + n_*\,(1 + \varepsilon_2 - a^2\varepsilon_2)\,. \tag{15}$$

Expression (14) also yields the existence of two elastic waves propagating with the same velocities, equal to the velocity of propagation of the transverse wave c_2.

The velocity V_1 is the velocity of propagation of a modified thermal wave accompanied by the thermal field, while the velocity V_2 is that of a modified thermal wave accompanied by the strain field. To elucidate the manner in



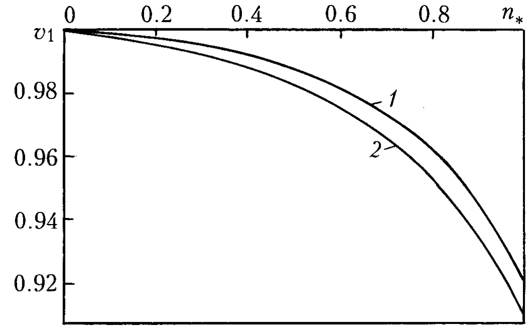


Fig. 2. Dependence of V_1/V_T (curves 1) and V_2/V_T (curves 2) on the parameter n_*: A, aluminum; B, lead.

Fig. 3. Velocity v_1 as a function of the parameter n_* for the same connectivity coefficient ε_1: 1) two-dimensional problem; 2) one-dimensional problem.

which the interaction of the process of elastic strain and the process of heat conduction affects the behavior of thermoelastic waves we consider the velocities of propagation of thermoelastic waves $v_{1,2}$ as functions of the parameter n_* (Fig. 1).

It follows from Fig. 1 that, when $\tau \to 0$, the velocity v_1 of the modified elastic wave tends to the velocity of propagation of the longitudinal elastic wave c_1. As the relaxation time of the heat flux increases, the velocity v_1 decreases as compared to c_1, i.e., the influence of the finite velocity of propagation of thermal disturbances leads to a decrease in the velocity of propagation of the longitudinal elastic wave. For low values of the parameter n_* the velocity of the modified thermal wave v_2 is much higher than the velocity c_1 of the elastic wave; as n_* increases, the velocity v_2 tends to a constant value insignificantly higher (≤5%) than the velocity c_1.

Let us compare the velocities of propagation of thermoelastic waves $v_{1,2} = V_{1,2}/c_1$ and the velocity of propagation of thermal disturbances $V_T = \sqrt{K/c_v\tau} = c_1\sqrt{n_*}$. The dependences $V_{1,2}/V_T$ for some materials from Table 1 are plotted in Fig. 2.

It follows from the behavior of the functions $V_{1,2}/V_T$ that the function of the velocity of propagation of thermal disturbances is an asymptote to the functions of the velocities of propagation of thermoelastic waves V_1 and V_2. Thus, whereas the velocity of the modified elastic wave tends to V_T with increase in n_*, the velocity of propagation of the modified thermal wave is approximately equal to V_T for low values of the parameter n_* and it increases (as compared to the velocity of thermal distances) with n_*.

In the case of the one-dimensional model of a generalized interconnected thermal-elasticity problem from (13) we obtain

$$v_{1,2} = \sqrt{(1 + 1/n + \varepsilon_1 \mp \sqrt{1/n + 2\,(\varepsilon_1 - 1) + n\,(1 + \varepsilon_1)^2}\,)/2}\,. \tag{16}$$

for dimensionless velocities of propagation of thermoelastic waves. Expression (16) yields the existence of two thermoelastic waves; v_1 is the velocity of propagation of a modified elastic wave and v_2 is the velocity of propagation of a modified thermal wave. The dependences of $v_{1,2}$ on the parameter n_* which have been plotted using (16) and the numerical data of Table 1 exactly coincide with the analogous dependences (Fig. 1 and 2) in the case of the two-dimensional problem. However if we take the coefficient $\varepsilon_1 = \varepsilon$ instead of the connectivity coefficient ε_2 in the two-dimensional problem, such a coincidence of the results is not observed. Figure 3 gives the dependences of the dimensionless velocity $v_1(n_*)$ for aluminum when the connectivity coefficients in both formulas (16) and expressions (15) are equal to ε_1.

The velocities of propagation of the modified elastic wave in the one-dimensional and two-dimensional interconnected problems of generalized thermoelasticity markedly differ in the case of one and the same connectivity coefficient ε_1 when $n_* \le 1$. The analogous behavior of the functions is observed for the dimensionless velocities of propagation of modified thermal waves v_2, which are determined by formulas (15) and (16) with a connectivity coefficient ε_1.

From the characteristic determinants (8), (11), and (13), it is quite easy to obtain bicharacteristics for the corresponding systems of equations of motion and to show that the velocity of propagation of the discontinuity surface $V = vc_1$ is the ray (radial) velocity of propagation of thermoelastic waves. We add that this equality does not occur in anisotropic media and the ray velocity is higher than the velocity of propagation of the discontinuity-surface front.

CONCLUSIONS

The employment of one and the same connectivity coefficient in dynamic problems of dissimilar dimensions in spatial coordinates leads to a distortion of the results of investigation of wave motion in both isotropic and anisotropic media. This is particularly important in generalized thermodynamics, since the relaxation time of the heat flux for metals is an extremely small quantity (its value is of the order of 10^{-11} sec) and has not been determined with a sufficient degree of accuracy.

NOTATION

λ and μ, Lamé constants; c_1 and c_2, velocities of propagation of longitudinal and transverse waves; ρ, density; β, thermomechanical constant; K, thermal conductivity; c_v, specific heat at constant strain; τ, relaxation time of the heat flux; T_0, initial temperature.

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